

# The unification of quark and lepton mixings: favored SUSY spectra for LHC

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The lower bounds on the masses of supersymmetric particles from LHC data and the upper bound on the sum of three active neutrino masses ( $\sum_i m_{\nu_i}$ ) from sky survey data have reached interestingly very strong limits. We show that there exists a correlation between the lower bounds on sparticle masses and the upper bound on  $\sum_i m_{\nu_i}$  if there is unification of quark and lepton mixings at the grand unification scale. The lower bounds on sparticle masses increase as the upper bound on  $\sum_i m_{\nu_i}$  decreases. For negative  $\mu$  (with conventional definition of sign) there is a strong lower bound on  $\sum_i m_{\nu_i} \gtrsim 1$  eV. Surprisingly, on the other hand, for positive  $\mu$  if  $\sum_i m_{\nu_i} < 1$  eV, the bounds on sparticle masses follow the recent LHC result. This upper bound on neutrino mass is strongly favored by the sky survey data and the positive sign of  $\mu$  is again preferred by  $g - 2$  of the muon. Finally, we find that the quark-lepton unification is consistent with present experimental data and it strongly constrains the parameter space of supersymmetry breaking models.

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*Introduction:* The unification of fundamental interactions [1, 2] in grand unified theories (GUT) motivates to study whether the origin of neutrino mass is from a common seed at the GUT scale and/or whether the origin of quark and the lepton mixing matrices are the same. Recently, the possibility that weak interaction properties of quarks and leptons parametrized by very different flavor mixing matrices at low energies may become identical at high energies (around GUT scale) has been shown in [3].

The quark masses originate from electroweak symmetry breaking (EWSB), while neutrino masses have different origin at the higher scale — the see-saw mechanism. They can be obtained at high scale theories using dimension-5 operator [4]. The energy dependence of the effective neutrino mass matrix below the scale where this operator is generated is described by its renormalization group equation (RGE) [5]. The Yukawa matrix close to the unit matrix is required to generate quasi-degenerate neutrinos [6], which is needed for large magnifications of solar and atmospheric mixing angles [7–9]. The required values of the neutrino masses can be tested at experiments, as the weak scale values of these masses to explain the oscillation data are in the range obtained from sky survey data [10] as well as from beta decays. Moreover, this range can also be accessible in future double beta decay experiments [11] and in KATRIN experiment [12].

Motivated by the above experiments on neutrino masses we study the unification of quark and lepton mixing matrices at the grand unification scale (where the gauge couplings unify). We consider the simplest supersymmetry (SUSY) breaking model — mSUGRA model.

The important feature of this work is that the running of neutrino parameters are exact as they are coupled with the running of minimal supersymmetric standard model (MSSM) parameters using the ISASUGRA program of the ISAJET package (V7.81) [13], while the supersymmetric contributions were approximate in the previous works discussed earlier. As a consequence, we obtain not only the more exact running of neutrino masses and mixing angles, but also achieve the ability to constrain SUSY parameters from neutrino parameters. Here, we consider two loop RGE for Yukawa, gauge and sparticle masses, and one loop RGE for neutrino parameters.

In this Letter we show that i) unification of quark and lepton mixings at the GUT scale leads to a correlation between upper bound on  $\sum_i m_{\nu_i}$  and lower bounds on sparticle masses at weak scale, and it constrains the SUSY parameter space and sparticle masses; ii) the present experimental results: lower bounds on sparticle masses from LHC, upper bound on  $\sum_i m_{\nu_i}$  from sky survey data and the sign of  $\mu$  from  $g - 2$  of muon from Tevatron experiment are consistent with quark-lepton unification.

*Renormalization group evolution:* The solution of the coupled RGEs for neutrino parameters along with SUSY parameters are obtained by an iterative cyclic process (weak-to-GUT and then GUT-to-weak) with GUT boundary conditions following the mSUGRA model [15–18]. The neutrino parameters are also set at GUT scale. The Higgsino mass  $\mu$  and the soft Higgs bilinear term  $B$  are fixed from radiative electroweak symmetry breaking (REWSB). This has been done by the following steps. First, we set the gauge couplings and Yukawa couplings at electroweak (EW) scale and run only these nine couplings up to GUT scale (where  $g_1$  and  $g_2$  meet) setting other required mass parameters at SUSY breaking scale with approximate values. Now, we set the GUT bound-

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ary conditions for neutrino and SUSY parameters; put  $\mu, B = 0$  (one can also put arbitrary values), and run down to weak scale. After adding the loop corrections to Higgsino mass parameters  $m_{H_u}^2$  and  $m_{H_d}^2$  we calculate  $\mu$  and  $B$  from EWSB condition. Taking these  $\mu$  and  $B$  values as well as the RGE evolved SUSY parameters and neutrino parameters, we run up to GUT scale and put the GUT values of  $\mu$  and  $B$  as they come and reset all other parameters at the GUT scale as earlier. We iterate this process until all parameters converge to a certain tolerance. The iteration are needed as the  $\mu$  and  $B$  are involved in the running of other parameters.

To understand the results the analytical formula for RGE of neutrino mixing angles and masses [5] are very important:

$$\begin{aligned} \dot{\theta}_{12} &= -\frac{Cy_\tau^2}{32\pi^2} \sin 2\theta_{12} s_{23}^2 \frac{|m_1 e^{i\varphi_1} + m_2 e^{i\varphi_2}|^2}{\Delta m_{21}^2} \\ &\quad + O(\theta_{13}), \end{aligned} \quad (1a)$$

$$\begin{aligned} \dot{\theta}_{13} &= \frac{Cy_\tau^2}{32\pi^2} \sin 2\theta_{12} \sin 2\theta_{23} \frac{m_3}{\Delta m_{32}^2 (1 + \zeta)} \times \\ &\quad \times [m_1 \cos(\varphi_1 - \delta) - (1 + \zeta) m_2 \cos(\varphi_2 - \delta) - \\ &\quad \zeta m_3 \cos \delta] + O(\theta_{13}), \end{aligned} \quad (1b)$$

$$\begin{aligned} \dot{\theta}_{23} &= -\frac{Cy_\tau^2}{32\pi^2} \sin 2\theta_{23} \frac{1}{\Delta m_{32}^2} \left[ c_{12}^2 |m_2 e^{i\varphi_2} + m_3|^2 \right. \\ &\quad \left. + s_{12}^2 \frac{|m_1 e^{i\varphi_1} + m_3|^2}{1 + \zeta} \right] + O(\theta_{13}). \end{aligned} \quad (1c)$$

where  $\zeta = \Delta m_{21}^2 / \Delta m_{32}^2$ ,  $\Delta m_{21}^2 = m_2^2 - m_1^2$ ,  $\Delta m_{32}^2 = m_3^2 - m_2^2$ ;  $C = 1$  in MSSM and  $-3/2$  in Standard Model (SM).

$$16\pi^2 \dot{m}_1 = [\alpha + Cy_\tau^2 (2s_{12}^2 s_{23}^2 + F_1)] m_1, \quad (2a)$$

$$16\pi^2 \dot{m}_2 = [\alpha + Cy_\tau^2 (2c_{12}^2 s_{23}^2 + F_2)] m_2, \quad (2b)$$

$$16\pi^2 \dot{m}_3 = [\alpha + 2Cy_\tau^2 c_{13}^2 c_{23}^2] m_3, \quad (2c)$$

where  $\alpha = -3g_2^2 + 2y_\tau^2 + 6(y_t^2 + y_b^2) + \lambda$  for SM; and  $\alpha = -6/5g_1^2 - 6g_2^2 + 6y_t^2$  for MSSM. We are using GUT charge normalization for  $g_1$ .  $F_1$  and  $F_2$  contain terms proportional to  $\sin \theta_{13}$ :

$$F_1 = -s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta + 2s_{13}^2 c_{12}^2 c_{23}^2, \quad (3a)$$

$$F_2 = s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta + 2s_{13}^2 s_{12}^2 c_{23}^2. \quad (3b)$$

From the RGE it is clear that one needs quasi-degenerate neutrino masses and normal hierarchical mass pattern for radiative magnification of the mixing angles; the other consequences are discussed later in context of explaining the results.

*Result:* We find the allowed points in parameter space by scanning randomly over the following ranges of the

parameters; common scalar mass ( $m_0$ ): 0.05 - 3 TeV, common gaugino mass ( $m_{1/2}$ ): 0.05 - 3 TeV, common trilinear coupling ( $A_0$ ):  $-3m_0 - +3m_0$ , sign( $\mu$ ):  $\pm 1$ , and  $\tan \beta$  (ratio of two vacuum expectation values of  $H_u$  and  $H_d$ ): 35 - 70. We set the ranges for neutrino masses  $m_1^0, m_2^0, m_3^0 = 0 - 0.7$  eV, neutrino mixing angles  $\theta_{12}^0 = 0.22(1 \pm x_1)$ ,  $\theta_{13}^0 = 0.0039(1 \pm x_2)$ ,  $\theta_{23}^0 = 0.034(1 \pm x_3)$ , CP phase  $\delta_{CP}^0 = 60^\circ(1 \pm x_4)$ , and Majorana phases  $\varphi_1^0, \varphi_2^0 = 0 - 360^\circ$ . We have chosen  $x_i$ s (uncertainties in the unification) randomly within the range  $0 \leq x_i \leq 40\%$  ( $i = 1, 2, \dots$ ) as an uncertainty due to approximate evolution [19] of CKM parameters. We have checked varying the upper limits of  $x_i$ s from 30% to 50% that the results do not change drastically; the bounds (discussed later) become gradually stronger as the upper limits of  $x_i$ s are decreased.

We set the experimental bounds obtained from LEP data:  $m_h > 114.5$  GeV,  $m_{\tilde{\chi}^\pm} > 103$  GeV [20] as these masses can be dominated by the value of  $\mu$ <sup>1</sup>. We consider only the points in the parameter space that can produce neutrino oscillation parameters at weak scale within the present global-fit ranges at  $3\sigma$  [21]:  $\sin^2 \theta_{23} \simeq 0.52^{+0.12}_{-0.13}$ ,  $\sin^2 \theta_{12} \simeq 0.312^{+0.048}_{-0.042}$ , and  $\sin^2 \theta_{13} \lesssim 0.039$ . The threshold corrections are added to the mass squared differences [22]. These are not significant in this scenario with normal slepton mass hierarchy. However, we have studied the unification with and without threshold corrections varying the ranges of mixing angles and mass squared differences. We present the plots for  $\Delta m_{21}^2$ :  $5 - 10 \times 10^{-5}$  eV<sup>2</sup> and  $\Delta m_{32}^2$ :  $2 - 3 \times 10^{-3}$  eV<sup>2</sup>, respectively, considering the threshold corrections. Obviously, more stronger constraints are obtained for narrower ranges of oscillation parameters (discussed later).

The generation of neutrino mixing angles at weak scale in the ranges allowed by global-fit of neutrino oscillation data needs large radiative magnifications and demands very high value of  $y_\tau$ . As these ranges at present are very narrow, it fixes  $y_\tau$  and consequently determines the value of  $\tan \beta$  as a function of the sum of three active neutrino masses ( $\sum m_{\nu_i}$ ) at the EW scale. As  $\sum_i m_{\nu_i}$  is lowered more higher value of  $y_\tau$  is required, which demands more larger value of  $\tan \beta$ . This is shown in the first plot of Fig. 1.

The solar and atmospheric mass squared differences are different by two order of magnitude as well as the magnification for solar angle is  $\sim 3$  and for atmospheric angle is  $\sim 20$ . To accommodate all these parameters in the experimentally allowed ranges for a given neutrino mass scale the Majorana phases are constrained in very

<sup>1</sup> If we withdraw the LEP bounds, the relatively lower sparticle masses are allowed, but the correlation of the bounds (discussed later) with  $\sum_i m_{\nu_i}$  remains.

narrow regions (see second and third plot of Fig. 1). This can be understood from Eq. 1a and from Eq. 1c.

The upper value of  $\tan \beta$  is bounded from REWSB or from the LEP bounds of  $m_{\tilde{\chi}^\pm}$  or  $m_h$ . As  $\tan \beta$  increases  $m_{H_d}^2$  decreases; and it can even be negative. This leads to smaller  $\mu$  at larger  $\tan \beta$ . At more higher  $\tan \beta$  REWSB becomes impossible as  $\mu^2$  becomes negative. In case of  $\mu < 0$  the loop corrections at EW scale make  $m_{H_d}^2$  more lower compared to  $\mu > 0$  and it gives bound on  $\tan \beta \lesssim 55$ . This restricts the increase in  $y_\tau$  and consequently leads to a lower bound on  $\sum_i m_{\nu_i} \gtrsim 1$  eV, which is very unlikely to be allowed by the present cosmological data [10].

On the other hand, for  $\mu > 0$  one can increase  $\tan \beta$  up to 65 leading to a decrease in  $\sum_i m_{\nu_i} \approx 0.6$  eV, which is very strongly favored by the neutrino data from sky survey [10]. The REWSB and the value of  $\mu$  also depend on value of  $m_{H_u}^2$  and  $m_{H_d}^2$  at GUT scale as well as on sparticle masses (through RG running and loop corrections at EW scale). Again, for a given  $\tan \beta$   $y_\tau$  depends on the value of  $\mu$ , gaugino masses and others through radiative corrections at weak scale. This leads to strong bounds on sparticle masses correlated with the upper limit of  $\sum_i m_{\nu_i}$ :  $m_{\tilde{g}} \gtrsim 1$  TeV,  $m_{\tilde{t}_1} \gtrsim 0.7$  TeV, and  $m_{\tilde{e}_L} \gtrsim 0.5$  TeV when  $\mu > 0$  and  $\sum_i m_{\nu_i} \leq 1$  eV. These results depend only on  $\sum_i m_{\nu_i}$ , other parameters are scanned over their whole ranges. Surprisingly, all these bounds follow the recent LHC results and are well above the LHC bounds [14]; and again, the neutrino mass limit  $\sum_i m_{\nu_i} \lesssim 1$  eV is strongly favored by the neutrino data from sky survey.

At this large value of  $\tan \beta$ ,  $m_{\tilde{\tau}_1}$  can be the lightest supersymmetric particle (LSP) in some cases depending on the choices of other parameters, mainly  $A_0$  and  $\text{sign}(\mu)$ . Another consequences of such high values of  $\tan \beta$  with positive sign of  $\mu$  are successful explanation of  $g - 2$  of muon [23]. Again, for positive  $\mu$ ,  $t - b - \tau$  unification is also possible [24].

In Fig. 2 we present neutrino mass correlated bounds in the planes of  $m_0 - m_{1/2}$ ,  $m_{\tilde{e}_L} - m_{\tilde{\tau}_1}$ ,  $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$ , and  $m_{\tilde{g}} - m_{\tilde{t}_1}$ , respectively. For each plot the allowed points are separated for three  $\sum_i m_{\nu_i}$  ranges:  $< 0.7$  eV,  $0.7 - 1.0$  eV and  $> 1$  eV, respectively, to show the dependence on the neutrino mass scale. As an example, in case of  $\mu > 0$ , for whole range of  $\sum_i m_{\nu_i}$  there is a bound on  $m_0 \gtrsim 1.5$  TeV only when  $m_{1/2} \gtrsim 0.4$  TeV; but, for  $\sum_i m_{\nu_i} \lesssim 1$  eV  $m_{1/2} \gtrsim 0.5$  TeV over whole range of  $m_0$ . In all these plots we randomly choose all the parameters and there is no correlation among them. So, these are only neutrino mass dependent bounds. From these plots one can easily find the values of individual sparticle masses and the differences  $m_{\tilde{e}_L} - m_{\tilde{\tau}_1}$ ,  $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$ , or  $m_{\tilde{g}} - m_{\tilde{t}_1}$ , which have definite pattern and one can predict the interesting possible collider signatures at LHC.

In case of other supersymmetry breaking scenarios one can also expect similarly strong bounds on sparticle masses from this unification criteria as one needs large  $\tan \beta$  and again as it requires larger  $m_{H_u}^2$  and  $m_{H_d}^2$  at GUT scale for EWSB. But, the above differences in the sparticle masses will then have different definite pattern as the GUT boundary conditions are different and the range of  $\tan \beta$  is very narrow. This may make the possibility in distinguishing the models.

*Conclusion and discussion:* In this Letter we have shown that unification of quark and lepton mixing matrices at the GUT scale leads to a very interesting correlation between the upper limit on  $\sum_i m_{\nu_i}$  and the lower limits on sparticle masses. This arises due to the fact that there exists a lower limit on  $\tan \beta$  for a given  $\sum_i m_{\nu_i}$  from quark-lepton unification. As  $\sum_i m_{\nu_i}$  decreases lower limit on  $\tan \beta$  increases. For a given high value of  $\tan \beta$  very strong lower bounds of sparticle masses appear from REWSB. As  $\tan \beta$  increases lower bounds of sparticle masses increase significantly. There is an upper limit on  $\tan \beta \lesssim 55$  for  $\mu < 0$  from REWSB and it constrains  $\sum_i m_{\nu_i} \gtrsim 1$  eV. For  $\sum_i m_{\nu_i} \lesssim 1$  eV only  $\mu > 0$  is allowed and there exists strong lower bounds on sparticle masses ( $\gtrsim$  TeV).

Interestingly,  $\sum_i m_{\nu_i} < 1$  eV is favored by the recent neutrino data from sky survey and the positive value of  $\mu$  with high  $\tan \beta$  is strongly favored by  $g - 2$  of muon. Again, for positive  $\mu$ ,  $t - b - \tau$  unification is also possible. Finally, we find that present experimental data are consistent with quark-lepton unification and it can constrain the parameter space of the supersymmetry breaking models.

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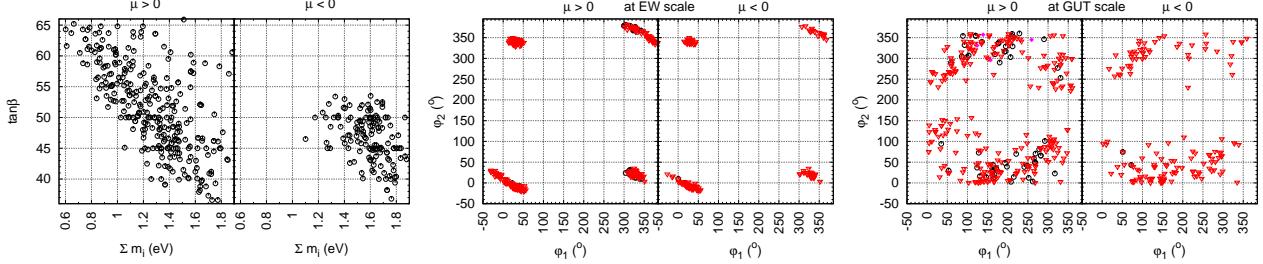


FIG. 1: The lower bound for  $\tan \beta$  as function of  $\sum_i m_{\nu_i}$  (first), and the allowed parameter space for Majorana phases in  $\varphi_1 - \varphi_2$  plane at the GUT scale (middle) as well as the weak scale (last) for both sign of  $\mu$ .

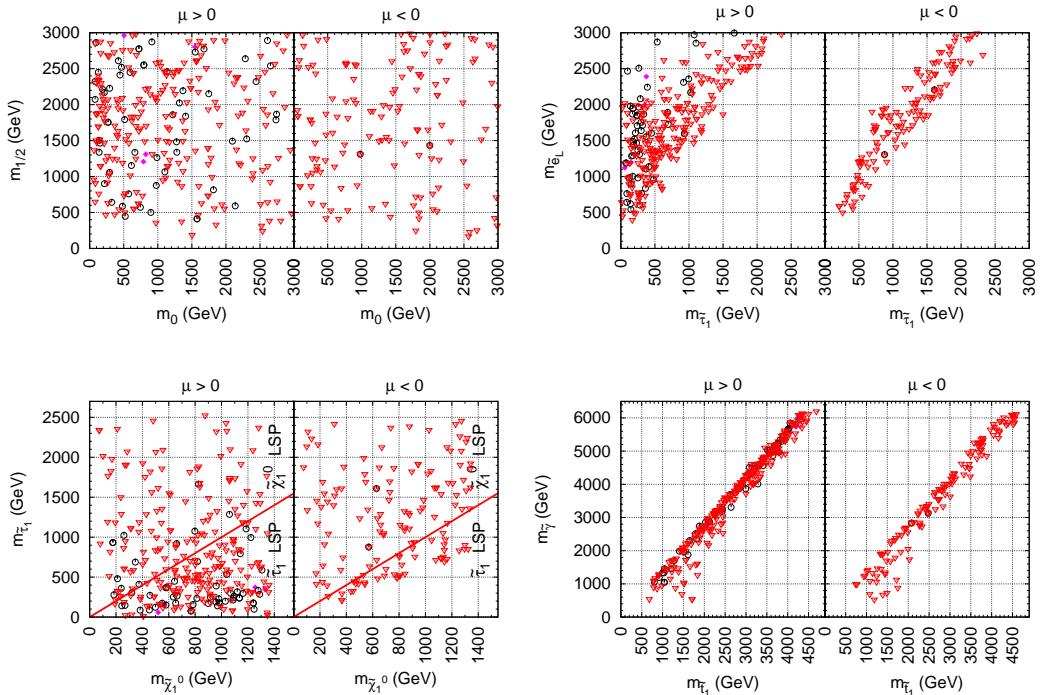


FIG. 2: The allowed region in the plane of  $m_0 - m_{1/2}$  (first),  $m_{\tilde{e}_L} - m_{\tilde{\tau}_1}$  (second),  $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$  (third) and  $m_{\tilde{g}} - m_{\tilde{\tau}_1}$  (fourth), respectively. The allowed points for  $\sum_i m_{\nu_i} < 0.7$  eV, 0.7 - 1 eV and  $> 1$  eV are represented by solid box, circle and triangle, respectively.

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